POTENTIAL AND PITFALLS OF JOINING TOGETHER COMPUTER ARTIFACTS (REPRESENTATIVES) TOWARDS A CONSTRUCTION OF MEANING IN SECONDARY SCHOOL ALGEBRA

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INTRODUCTION

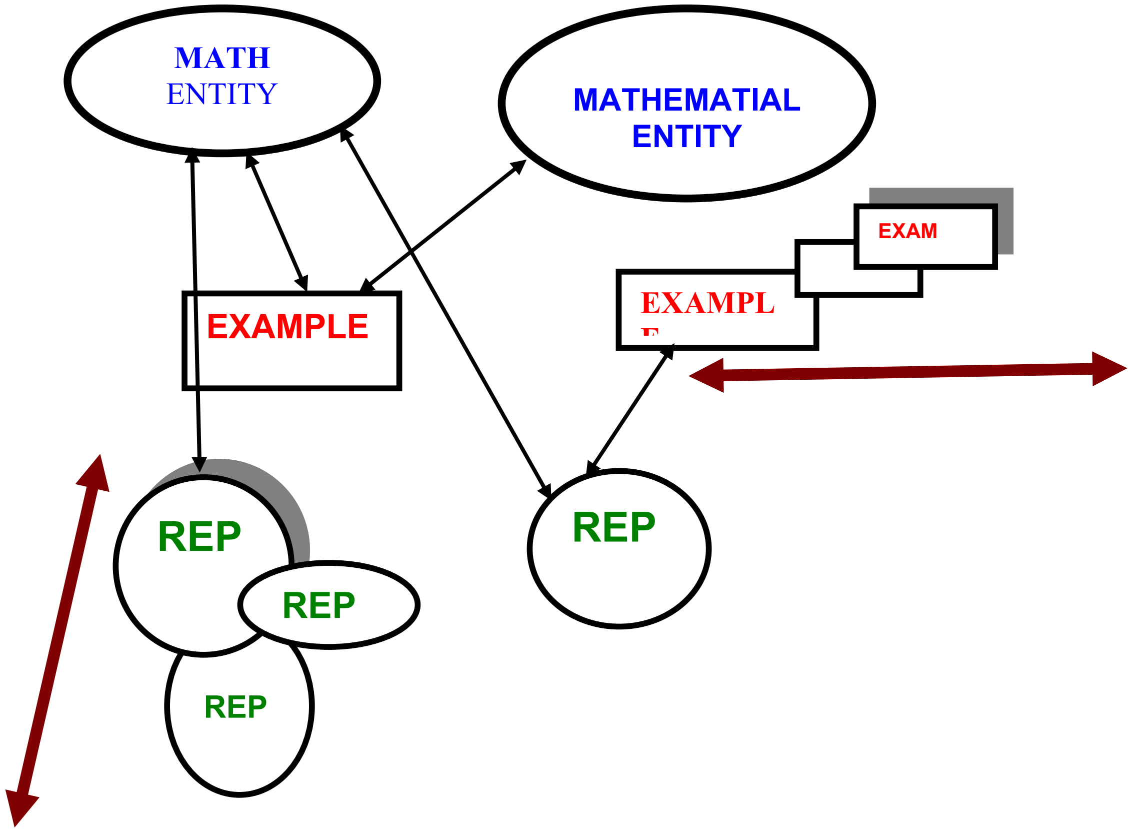
On the relationships between a mathematical entity, and its examples and representatives

Mathematical entities, such as the function concept or geometrical concepts, are constructs that have been elaborated by the mathematical community. These entities are not directly accessible to students but are known through their properties, which are the invariants of this mathematical entity across their possible instances. But these invariants can be approached only through representatives, which are examples, or parts from examples, about which students and teacher talk. Similarly, direct manipulations on mathematical entities are impossible. Manipulations are possible on examples or representatives only. Like words in verbal interaction, representatives are an inherent part of communication in mathematical activity because the conceptualization of the mathematical entity is constructed by actions on them.

As examples of a mathematical entity, representatives may often be ambiguous. Examples contain all the invariant properties of the concept (also called the *critical attributes* of the concept). In addition to the invariants of the mathematical entity, examples have also self-attributes, which might change from one example to another. In some cases, students identify such self-attributes as critical attributes of the concept. This is in cases where the example is the only one the student is familiar with or the one the student prefers over other examples to reason about the concept (the example is then called a prototype). The self-attributes of the example/prototype are then imposed on all the other examples. . Examples are then ambiguous regarding the concept they refer to. (Hershkowitz, Ben-Chaim, Hoyles, Lappan, Mitchelmore, & Vinner, 1990).

As parts from examples, representatives may be often ambiguous, in the sense that only some of the critical properties of the entity are displayed in the representative. It may lead to two different phenomena: (i) difficulty in constructing or seeing the mathematical entity through its partial representative; (ii) linking a representative to another entity with critical attributes compatible with the representative. Hence, the same representative may refer to more than one mathematical entity. For example the same graphical representative can refer to two different functions. The relationships between the mathematical entity and its examples/ representatives are described in Figure 1.

Figure 1



Computerized tools and representatives.

A common claim is that computerized tools can play an advantageous role in assisting students to make connections between and within various representations of the same mathematical entity (e.g., Kaput, 1992).

The most common visible outcomes of actions mediated by multi-representational tools are also the representatives (Schwarz & Dreyfus, 1995). Representatives of mathematical entities, mediated by computerized tools are; specific "windows" for graphs, or specific tables of values as representatives of a function; specific drawings as representatives of a geometrical figure.

The ambiguity and the power of the computerized tool encourage the production of various representatives upon need, and also stimulate students' requirement for and ability with interweaving representatives together. In so doing, students may extract the invariant properties of the mathematical entity and thus overcome the ambiguity within the single representative. This process is part of the students' search for mathematical meaning. On the other hand, technological environments can induce students to reach "false representatives" (those that do not represent the critical properties of the mathematical entity at all) or to interweave representatives together in a non-meaningful, algorithmic fashion.

In the following we will show two case studies which demonstrate the mediating power of representatives in learning the topic of functions.

THE FENCE CASE STUDY

We document several praxes in a parochial school Grade 9 class studying an introductory course on functions. It consisted of a sequence of tasks (for small group problem solving or individual assignments) organized around problem-situations. Students had graphic calculators (TI-80) at their disposal. Many interactions took place in the classroom: among students in small groups, between the teacher and students, or between individuals/groups and computerized tools. Typically, activities consisted of a few phases: (i) problem solving (in groups of four); (ii) the writing of group reports on the ideas raised during the problem solving; (iii) synthesis in the form of debate orchestrated by the teacher; (iv) a homework assignment, designed by the teacher and based on ideas raised in the group reports. When students (as individuals or in small groups) reported on their work, they knew that the teacher would use some of their reports to discuss issues further in the class. The teacher was explicit about the fact that writing accurate reports eventually helps the class to learn better.

From the very beginning of the year, students were explicitly invited to use the graphic calculators to enter algebraic formulae, draw graphs and walking on them, construct tables of values and to pass from one representation to another, in other words to produce numerous representatives. Students were encouraged to decide which representation/s to use, when and how to link them, and in which medium to work (e.g., paper and pencil, graphical calculator, discussion in teams, etc.). Such practices were first "imposed" by the teacher, but quickly became students' autonomous practices.

The Fence was given at the beginning of the year as a group evaluation task, after the three first sessions of instruction during which students performed a series of tasks on functions with graphical calculators. Students manipulated representatives to investigate situations, passed to new representative in a new or same representation (what we call "*to interweave*" representatives) when they felt it was needed. The formulation of the Fence was then given to the students:

Oranim school received a 30m long wire to fence a rectangular vegetable garden lot. The lot is contiguous to the school wall, so that the fence has three sides only (see Fig. 3).

1. *Find four possible dimensions for the lot, and the corresponding areas.*
2. *For which dimensions does the lot have the biggest area?*
3. *If one of the dimensions is 11m, what is the area of the lot? Can you find another lot with the same area? If yes, find its dimensions, if not, explain.*
4. *How many lots with the following areas are there: 80m^2, 150m^2?*

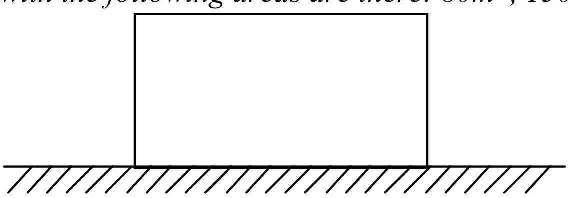


figure 3

Problem Analysis:

Question 1. was aimed at familiarizing students with the situation through calculating four different possible dimensions of the lot. For example, when one side of the lot is 12m there are two possibilities: If such a side is parallel to the school wall then the other side is equal to 9m [(30 – 12)/2] and the area of the lot is 12*9 = 108m^2. If the side perpendicular to the wall is 12m, then the other side is 6m (30 – 2*12 = 6), and the area of the lot is 12*6 = 72m^2.

Question 2. was meant to be solved by walking on the graph of the function y = x(30 – 2x), where x represent the length of the perpendicular side, and finding the values of the "highest' point. Later on during the year, when students learn to find the maximum or the minimum of quadratic functions, they are able to tackle this problem algebraically. This question is a very good example of the mathematical power that students can gain from using graphical calculators before they acquire formal knowledge of a topic mastered through mathematical procedures.

In question 3. students may again capitalize on the possibility to walk on the graph and read the coordinates of the points. Such a "walk" is intended to lead the students to realize that there are lots with different dimensions but with the same area.

In question 4. students are induced to realize that there are values of areas that correspond to no dimensions of the lot.

Eight groups of four girls solved the Fence, resulting in eight group reports. One week later, the teacher gave a homework assignment on the basis of the group reports. 32 individual homework assignments were collected and analyzed. The reporting and the whole design were communicative actions by means of which representatives were shared by the whole class. In the following, we study how the teacher took advantage of the ambiguity of the representatives as artifacts to design homework assignments in which the individuals constructed the meaning of mathematical objects.

THE ROLE OF COMPUTER ARTIFACTS IN THE FENCE

a. Representatives-artifacts used in the group reports following the problem solving session

Diverse strategies were evoked in the group reports. For example, several groups evoked an inductive strategy that led to generalization: they organized their numerical examples of the dimensions of the lot as a table with three entries per row: first side, second side, area. Then, they generalized the numerical values in algebraic terms: x, 30-2x, x(30-2x). In other reports, students evoked a trial-and-error strategy, constructing a table and finding the maximum by successive approximation. Yet other groups evoked a modeling strategy: they first constructed a formula and used the calculator to draw a suitable graphical representative and read the maximum of the function by walking on it. The diversity of strategies evoked was accompanied by an even more diverse collection of representatives and by various strategies for interweaving them. The following is one example from the group reports (more examples may be read in Schwarz and Hershkowitz, 2001):

Example: This excerpt exemplifies a group who modeled the Fence situation by an algebraic formula. This group was "in a hurry" to find a formula, inserted 30 – 2x (the formula for the second side) as the formula of the area, and reported on the surprise caused by the graphical representative obtained (the text written by the group is italicized to differentiate it from our observations):

> *We drew the graph on the graphical calculator, according to the formula 30-2x, with a range of 0-15 for x and a range of 0-150 for y* (Fig. 4a).

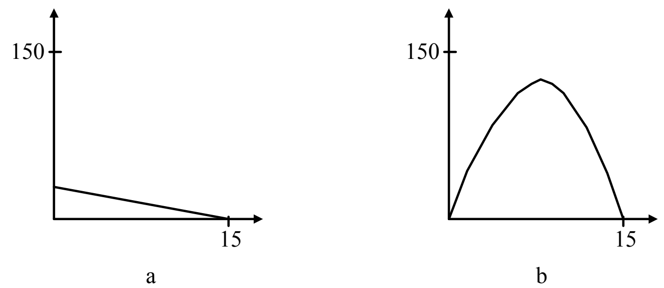


figure 4

> *It* (the graph) *did not seem OK because we had to find the area and the formula fitted the second side; so we understood that the formula was wrong and we decided to replace it by (30-2x)x because this is the area formula* (two arrows pointing to x and to 30-2x and labeled first side and second side, appeared in the group report). *We looked for the largest area. We got a graph that seems to us a more correct one* (Fig. 4b). *Note: we did not change the range, only the formula.*

It appears that the students were surprised by the first graphical representative they produced. So, they turned back to the algebraic representation, and corrected the formula to obtain what they called a *"more correct graph*". It seems that the students rejected the first two representatives because the second one (the graph) did not make visible the intrinsic attributes of their mental representation of the situation. Consequently, they produced two more representatives and accepted the second one, because it made visible the requested property of the corresponding function, having a maximum at some point. This example seems to show that students link representatives to the function they are meant to represent, through actions of *interweaving* representatives and rejecting those which are not compatible with their concept image.

b. Transformation of artifacts in the individual homework assignments.

As described above, all the representatives were collected by the teacher, who chose some of them as artifacts for subsequent homework assignments. She collected the reports and "redistributed" some of them in the homework assignment. For example, the teacher used the representatives in Figure 4a in the following question:

A few students drew this straight line as the area graph (see Fig. 4a). *Most students said that this graph does not fit the area. Explain why. Give as many reasons as you can.*

This question could be answered directly and quite simply by invoking only local properties of the linear graph. For example, it is clear that when x = 0, the area is 0, but for the linear graph, when x = 0, the area is not 0. However, only 7 students gave such answers. The remaining 25 students preferred *to compare* the linear graph presented in the homework sheet with representatives they had already constructed. A typical justification of the incorrectness of the linear graph is the following:

I know that for the side zero, there is no area. Also for the side 15, there is no area. So I know that between zero and 15, there must be an arc ... and so for 1, 2, it increases until it must go down in order to get to 15 (where there is no area). And on this graph, (Figure 4a) *it always goes down.*

In her group report, this student and her peers had first constructed a table of values (with 0 and 15 as x-values), then modeled the formula of the area, and drawn the graph. She could have used this graph as a whole to dismiss the proposed linear representative. Instead, she used the properties represented by the representative produced by her group, to show that the linear representative does not have the required attributes: "her" graph showed no area for the two extreme values x=0, and x=15, a fact that matches the Fence. Thus, such a graph must increase and then decrease. It may seem strange that the student used such a sophisticated justification. But, students were accustomed to handle ambiguous outcomes-representatives whose meaning was clarified by comparing them with other representatives, and by deciding whether they refer to the same meaning. This example shows an additional type of *interweaving of representatives* leading to constructing mathematical meaning - by comparing them.

In sum, when the teacher found the students with the linear graph, she created a state of intersubjectivity in which the ambiguous linear representative led students to ask whether it refers to the same state of affairs as when they solved the problem in their groups. The decision relied on a property (critical attribute) of the function, the domain of increase/decrease. It appears then that the teacher's selection of an artifact produced by groups in the class, and its distribution led students to cope with a mathematical construct.

COMPUTER ARTIFACTS AND CONSTRUCTION OF MEANING

This case study has shown that representatives produced via the mediation of the calculators, helped in the social construction of meaning of functions. We showed that, although the relations between concept and examples, examples and representatives, or concepts and representatives, are inherently ambiguous, dealing and relating to many examples or acting on many representatives did lead to the construction of the meaning of mathematical entities. We showed that *representative ambiguity* the *production* of various representatives and the need and ability to *interweave* them together, helped to extract invariant properties of functions. We also showed that the plethora of examples and representatives produced by the graphic calculators upon need, and the actions facilitated by them to produce new representatives *led students to clarify ambiguities*. The computerized tool enabled students to compare them, to produce new representatives and to integrate or reject (partial) representatives. Advantage was taken of partiality *to view the intrinsic attributes* of the function by *refining* through interweaving representatives (e.g., by passing to another representation and/or by manipulating on representatives in order to reduce ambiguity).

Resolving ambiguities was not only an outcome of the manipulation of computer representatives. The teacher had a central part in leading students in a dialectical activity leading to the resolution of ambiguities. Figure 8 shows the different phases of the Fence from the perspective of the teacher. At a first stage, groups of four students (shown as "clouds") solve the Fence and report on their actions during the solution. The reports include numerous representatives. Some of these representatives may reflect "correct" actions (such as the algebraic representatives $y = x(30 - 2x)$ and $y = x(30 - x)/2$ and all graphical representatives derived from these formulae), some may reflect incorrect ones (such as the formula $y = 30 - 2x$ and the linear graph obtained). As they are the footprints of the idiosyncratic moves undertaken by each of the groups, most of the representatives are different in form. A short analysis of the reports (as in the example above) shows that most of the groups coped with the diversity/ambiguity of the representatives to solve the Fence.

The role of the teacher in the home assignment was indirect. After collecting the representatives in the reports as raw material to take advantage of, the teacher selected three of them having the potential to lead to the construction of further meaning. It was clear to all students that these representatives were private outcomes of particular groups in the class.

The teacher first imposed an implicit community by forming groups of students at the beginning of the year to solve problem situations. She supported reporting and criticizing as *communicative actions* that gave the actors a visible role: The teacher redistributed some of the representatives in the homework assignment, to be the property of the whole class. Although we do not report here on the actual interactions that took place in the class during the Fence activity, we suggest that the class began then to function as a new community whose overall motive moved gradually from reaching the solution to reflecting on acceptable solutions.

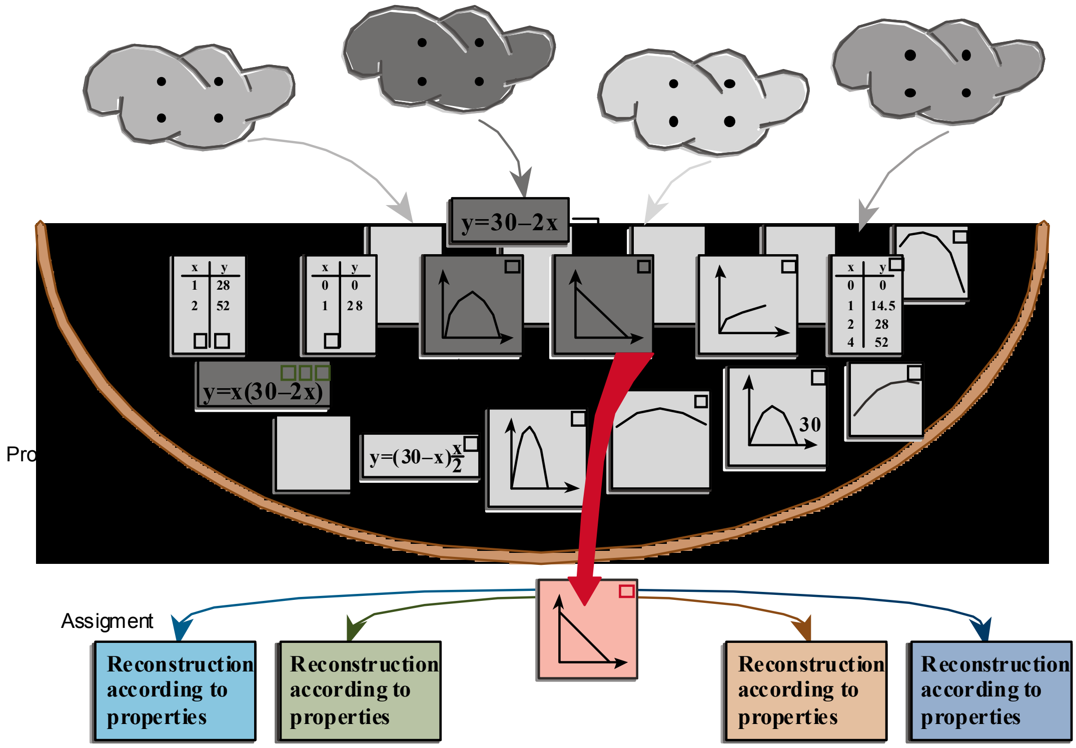


figure 8

This study has shown the beneficial role of ambiguity in the learning of the function concept, in the presence of computerized tools. Two natural questions stem from the present study. The first one concerns deepening the cognitive mechanisms that occur when one copes with ambiguity. The second question concerns the generalizability of the findings of the present study. The second case study research will put some light on these questions.

THE GROWING RECTANGLES CASE STUDY

In designing mathematics learning with the mediation of computerized tools, one of the crucial questions to be considered is how much and in what way we would like the tool "to do the work" for the students. In problem situations where the solution is achieved mostly via graphical representatives, and algebraic models are mostly used as "keys" for obtaining graphical representatives on the screen, the algebraic representatives and their form seem minimized in importance, and students may tend to generate them from tables by mechanistic-algorithmic procedures. The above questions will be mainly demonstrated within the description and analysis of a case study involving a group of three 10th graders (about 16 years of age) in a Montreal high-school, working together to investigate and solve a problem situation on the topic of functions, having graphing calculators (TI-83 Plus) at their disposal. The role of contextual factors is highlighted by means of contrasts with the work of students on the same problem from another country (The same class which was analyzed at the first research above).

The protocol analysis of the group work will reveal a dialectical problem-solving process that develops between two ways of making use of the computerized tool: a mechanistic-algorithmic one and another one that is led by students' search for meaning.

Moreover, we tell the story of the use of two kinds of representatives in the process of problem solving: representatives that do not represent the properties of the mathematical objects involved at all, and representatives that do.

The analysis of the group activity is described in four rounds. On the whole, the investigative activity done by this group and their solution strategies are monitored by their need for a search for meaning. But, in Rounds 1 and 2, the process of "joining together (interweaving) tool-based representatives," which the students engage in, is mostly of the first kind, the mechanistic-algorithmic, which leads students towards some "false representatives." Rounds 3 and 4 provide evidence of a different way of joining together representatives, one that is clearly characterized by a search for meaning. The progress of the group from Rounds 1 and 2 into Rounds 3 and 4 is led by critical thinking and is supported by the students' control of the "joining-together-representatives" functions, which are part of the strength of the graphing calculator.

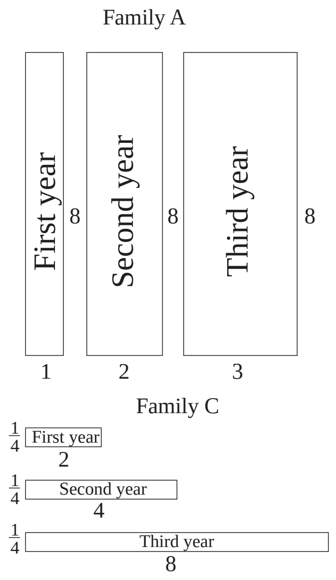
The Problem Situation and Some Comments About Its History

The following are the parts of the problem situation that are relevant to this paper**.**

Growing Rectangles

Each of the following three families of rectangles has its growth pattern:
*In family **A** the width grows each year by one unit; the length remains constant at 8 units. In **B** the width and the length of the rectangle grow each year by one unit. In **C** the length doubles each year, and the width remains equal to 1/4.*
Please investigate the problem in groups. At first try to generate various hypotheses concerning the following questions:

1. Please compare the areas of the 3 families of rectangles over the years. What are their initial situations? Which family (or families) takes over the other families (or family) and when?



2. *In which years will the area of each family exceed 1000 square units?*

Now check your hypotheses with mathematical tools (the help of the graphing calculator is recommended). Try to be as accurate as possible.

3. *Please write a report as a common product of your group.*

Try to describe your conjectures and what they were based on. What kinds of debates did your group have?

Try to describe the ways in which you solved the problem and in what ways you were using the graphing calculator

This activity was first tried in an introductory one-year-long course on functions with the mediation of graphical calculators (TI-81) in Grade 9 in Israel (for details see Hershkowitz & Schwarz, 1999). It took place during the sixth week of the course. At that stage, students had practiced the actions and passages between and within representations and representatives. They were aware of the fact that obtaining a graph representing a given phenomenon required an algebraic model.

The students in the Israeli class were first invited to suggest hypotheses without using the computerized tool, then to use it to check them. Students tried to figure out which family would eventually take the lead by using intuition and/or by computing "by hand" the areas of the three families of rectangles for a few values. Then students (in most of the groups) translated the situation into algebraic representations (with some difficulties in generating the function-area for Family C, $y=1/4*2^x$), and then obtained the graphs with their graphical calculators (see Figure 9 below for a stylized representation of the graphing window). After the groups had finished, the teacher discussed with the class the mathematical findings and strategies. Students reported that in the eighth year (x=8), the three rectangles had the same area, and that from that year on, Family C took the lead from Family A. Family B remained in between. The evidence provided by the different representations was accepted even if, for some students, it was unexpected; no student declared the computer wrong. Nevertheless, they tried to reinterpret the situation, and even to overcome wrong intuitions, by matching together representatives from different representations; the algebraic, the numerical, the graphic, and the phenomenon itself.

Figure 9

The sociomathematical norm of what constitutes evidence in problem situations was formed here as a consequence of students' interactions with the tool.

Before setting off to work in groups of threes on the problem situation, the research class from the Montreal school was asked also to read the problem and to vote on whether they thought that Family A, B, or C would have the largest area over the long term. A couple of students voted for Family A; most voted for Family B; and a few abstained. No one predicted that Family C would eventually have the largest area. Teams then started to work on the problem situation. They also had to produce a written report on the problem-solving process, as a common product of the team, and then to present it orally in front of the whole class during the concluding discussion. The whole activity was two periods long (an hour and a half in all).

In the following we will tell a short version of the group story. For more details see Hershkowitz & Kieran, 2001.

Round 1: Making the technological tool generate the algebraic expression for the situation.

The team (Kay, Sam and Ema) started from calculating a table of values, in order to be able to calculate the global differences between the 1st and 10th values, and checking their initial hypothesis as to which Family had the largest area over the long term. The calculations for Families A and C were done in a recursive fashion. The calculations for Family B were based on an explicit generalization for x (the year number).

Than the team had the objective - to draw the graphs in order to be able to compare the three Families by having intersection points. They inserted a few values of each Family from the table into the graphic calculator and got to Algebraic Model, by **linear regression** on the calculator. So they got:

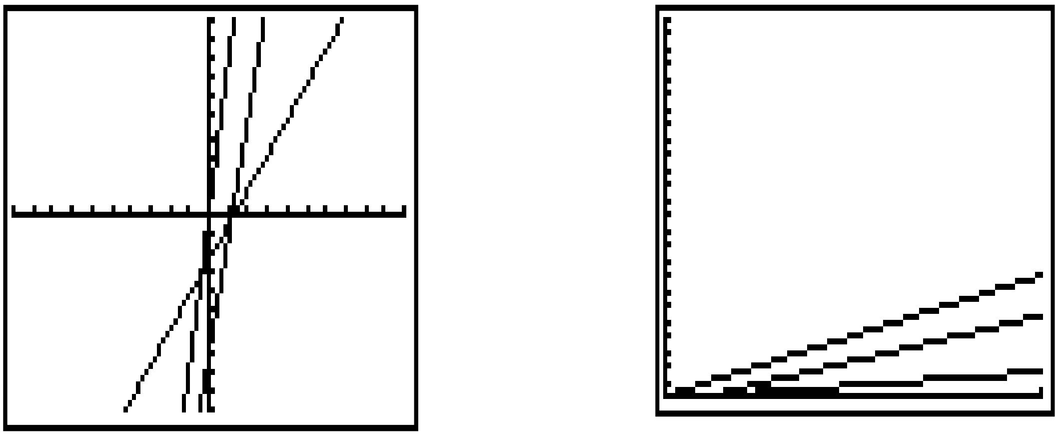
8x for family **A**; **6*x* – 7** for family **B** and **1.8*x* - 2.3** for family **C,** and than they received the following graphs on the calculator screen.

There are a few questions which might be asked at the end of this round:

1. Why did the group mechanistically use the calculator to generate and to join together the representatives (the numerical and algebraic representatives) for answering the question "WHEN?"
2. Did they consider the graphical representation more reliable?
3. Is a functional algebraic rule, and its joined-together graph, a stronger argument than mere numerical evidence?
4. Were the students just carried away by the algorithm itself?

Round 2: Failure at getting the technological tool to work for them: a turning point.

From the beginning of applying the regression option of the calculator tool and up to this moment, the group had appeared to be marching along a mechanistic path in a kind of automatic fashion, without doing very much in the way of reflection. When the graphs they generated did not look as they had expected them (Figure 10a), They changed the scales immediately Figure 10b. But, that did not help them to obtain the meeting point they wanted to see (see Figure 9). Repeated scale changes (even up to 1500 years) did not produce the elusive point of intersection that they had intuitively been expecting. They realized that there was no point of intersection in the given window and confirmed for them what had been evident from their reading of the graphs, graphs that had been produced by the expressions of the linear regression option. Yet, there was a sense of unease. They had failed in their attempt to get the technology to work for them. They had expected a single point of intersection somewhere in the first quadrant. It was the starting point of critical thinking.



a b

Figure 10

Round 3: A shift of attention.

This round starts with Kay beginning to think more about the situation: *"We try to find how much it increases, which one grows the most."* They returned to the text of the problem situation and reread it: "*Which family (or families) takes over the other families (or family) and when?"* Kay emphasized the *"AND WHEN."* Sam began to look at the paper-and-pencil table of values and said: *"We already figured out who is going bigger, but we can't answer when, because ..."* Then Sam and Kay noticed in the table that, at 8 years, all three families had an area of 64, and so Sam asked three times: *"So why aren't they meeting?"* They now paid attention to the hard numerical evidence that there was a single point of intersection. Their inability to match this evidence and the graphical representatives they had obtained became very clear and waited to be resolved and explained.

Kay substituted 8 into their algebraic expressions for Families B and C and obtained 41 and 12.1, instead of the desired 64, and she said: *"We did something wrong."* In an attempt to make sense of what was going on, the students restarted the process of joining-together-representatives, again with the regression option of the calculator tool. But, this time they were equipped with the critical thinking they had developed from the above comparison of the numerical and graphical representatives.

Sam began to systematically substitute values into the expression for Family B:*"6(1) - 7 is -1: that's wrong; it's not having a negative area*." With his substitution of 2 into the same expression, he became even surer that they were wrong. Kay wondered aloud: "*So do we make up our own equation; maybe it's not a linear regression."* Sam, who was still substituting, said, *"We're getting further apart; even C goes off."*

In this round, their mechanistic approach to joining-together-representatives was being put into question by a more meaningful one. It is not clear if they started to suspect the mechanistic routine itself or the way they had performed it. However, as will be seen in the next round, rather than abandoning the process carried out with the technology, they insisted that it be made to work for them.

Round 4: Insisting that the technology be made to work for them.

They start experimenting with other forms of regression available on the calculator tool:

Kay applies cubic regression for Family B and gets: $y = x^2$ [neat parameters and a correlation coefficient of 1].

Sam reacts: "*I don't know why I didn't figure it out from the beginning. I said it, why didn't I see it?*"

Kay tries other regression choices for Family C. The exponential regression leads to:

$y = 0.25 \text{ x } 2^{x}$ [correlation coefficient of 1].

Kay & Sam were a little uncomfortable with it.

Sam: "*We should see that they are all meeting at 64.*" [He draws the graphs on the calculator and gets the intersection point, visibly pleased].

The technology had now been made to work for them. The graphs, the equations, the paper-and-pencil table of values, and the situation all fit together. Doerr and Zangor (1999) have emphasized the importance of leading students to "develop a reasonable skepticism about calculator-generated results" and encouraging the establishment of classroom "norms that require results to be justified on mathematical grounds, not simply taken as calculator results" (pp. 271-272).

It is noted that these students did not throw aside their calculator tool when the graphs it produced could not be justified on mathematical grounds; they continued with it until it could be made to deliver correct mathematical representatives.

Discussion

As a fifth round the group was also asked to report on their flow of actions in this activity. reporting proceeded from an invitation expressed explicitly in the problem situation. How much their report reflects the real actions of 'joining together representatives' which were evidenced in the four rounds above? Those that were controlled by searching for meaning and those that were dominated by the presence of the algorithmic way to join the tool based representatives together? We can say (for details see Hershkowitz & Kieran, 2001) that on the whole students reported their actions as if they were controlled mainly by searching for meaning.

In short, we may conclude that the group completed their problem solving process by means of actions involving "joining together representatives" which, at the starting point as well as at the end, were controlled by the need to have meaning. But the sequence as a whole was an intertwining one where both the mechanistic and the meaningful were dialectically connected. The regression routines were the mechanistic parts of the sequence. The to-ing and fro-ing between the mechanistic and meaningful joining-together-of-representatives was at times characterized by lengthy segments of mechanistic activity. Nevertheless, the search for meaning always prevailed.

As mentioned before, when we speak of computerized tools in learning we usually speak of their "positive" potential in mediating learning (see the first case study above). The above example showed that such mediation might raise dilemmas for learning. A crucial dilemma is how much and in what way we would like the tool "to do the work" for the students. And more specifically, do we value that students be able to express a problem situation with algebraic models, and that they produce themselves this algebraic model?

Both classes in the "growing rectangles activity" were driven by a search for meaning in comparing the growth of the three families and were looking for the three graphs and their intersection points, knowing that the algebraic models were the keys for obtaining the graphs. The first class, which had a more limited tool (TI-81), started immediately to construct the algebraic models of the three families from the problem situation itself. This was not so easy for Family C and a few groups in this class failed. Our group from the second class trusted from the beginning the mediation power of their more advanced tool (TI-83 Plus) to do the work of generating the algebraic models for them. But in the first round they failed to get the right algebraic models, even for Family B. During that stage, they had used the tool in a mechanistic way, without engaging critical thinking, and thus obtained "false representatives."

What is the source of the differences between the two classes? It is obvious that the tool itself may explain part of the difference because the TI-81 has only a quite limited regression option. But, the dissimilar history of mathematics learning in the two classes is likely a major contributing factor. As we mentioned above, our second class had the experience of dealing with real world problems with non-idealized data, which usually do not fit perfectly an algebraic model. This encourages the use of regression techniques as a means of obtaining an algebraic model. So the students imposed the same kind of modeling technique on the "Growing Rectangles" problem situation. For students who have difficulty in modeling the growth of an exponential function from the situation itself, such as was experienced with the Family C rectangle, this technique may serve as a temporary scaffolding. Students may rely on a similar kind of scaffolding when using spreadsheets for modeling. (We have observed younger students investigating similar problems of exponential growth with EXCEL (Hershkowitz, 1999)).

The other side of this coin is that this scaffolding may stay longer then we, as mathematics educators, would like in the process of learning algebra. We had examples of students working on a problem situation in which exponential growth was investigated, where students were quite close to generating a closed-form exponential formula. But, when they discovered the EXCEL option of generating a recursive expression + dragging, as a quite efficient alternative for obtaining a whole set of data for the phenomenon, they curtailed their efforts to mathematize the problem situation in a higher level manner. While it might be argued that these students were engaging in a kind of algebraic thinking (Kieran, 1996), might the use of computerized tools in learning algebra reduce students' needs for high level algebraic activity?

In addition, we face an even more crucial dilemma; the algebraic representation and its form seem minimized in importance for students. Our case-study group, like groups in the other class, knew that the algebraic formula was the key, but perhaps because of their more advanced tool, and because of their learning history, the shape of the algebraic model seemed unimportant. In fact, students can now go from entering lists to a graphical representation without ever seeing or having to examine the algebraic representation of the situation. Does the use of these tools in algebra signal the beginning of the loss of the algebraic representation from our mathematical classes at the secondary level?

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